Doubling vs. Constant Bets as Strategies for Gambling

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Some gamblers use a doubling strategy as a way of improving their chances of coming home a winner. This paper reports on the results of a computer simulation study of the doubling strategy and compares the short term and long term results of doubling to gambling with a constant sized bet. In the short term players using a doubling strategy were more likely to win, then lose, however in the long term, the losses suffered by doublers were much greater than that suffered by constant bettors. It is argued that the use of a doubling strategy is related to an incomplete conceptualization of random events sometimes known as the 'law of averages.' A second simulation examined the fate of doubling in an ideal world in which the 'law of averages' was actually true. In this ideal world, doublers were much better off than constant bettors. The relationship of the results to a naive conceptualization of random events is discussed.

People like to gamble. For example, in recent surveys between 70% and 80% of the general population of Ontario consider gambling exciting (Ferris, Stripe & Ialomiteanu, 1996; Turner, Ialomiteanu & Room, 1998) and over 80% of the general public participates in some form of gambling. An increasing number of lotteries and casinos feed the public's appetite for gambling. Most people treat gambling as a

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form of entertainment that sometimes pays for itself. However, a small percentage of gamblers become addicted to gambling and experience large financial losses as a result.

There are many strategies that have been proposed to help a gambler beat the odds at games that involve pure chance (see Arnold, 1978 & Wagenaar, 1988 for critical reviews). Many of these systems are based on faulty assumptions regarding the nature of random probability. For example, a player might observe a roulette wheel and if a ball has landed on red 5 times in a row, he or she might place a bet on black in the belief that black is more likely to occur because on average black and red occur equally often. This is an instance of the representativeness heuristics (Kahneman & Tversky, 1982), and is sometimes called the law of averages (Arnold, 1978). It is important for gamblers and clinicians to understand these systems and their short coming since a majority of pathological gamblers in treatment claim to rely on their skill or on systems at least some of the time (Toneatto, 1998). Furthermore, there are significant correlations between measures of problem and pathological gambling and the self reported use of systems in both clinical samples (Toneatto, 1998) and in general population samples (Turner & Ialomiteanu, 1998, May).

There are several versions of the doubling strategy. For example, in the 'small' Martingale system for roulette, a person bets one unit on black or red. If the unit is lost, the bet is doubled. In roulette, doubling is only practical on simple bets (black, reds, odds or evens etc.) or other combinations of bets with odds near 50/50. Doubling with single numbers would not be possible because 2, doubled 37 times would exceed 137 billion. Thus all the remaining discussion will be about bets on blacks, reds, odds or evens, which pays even money and the player wins roughly 1 time for every 2.1 games played. If there is no house limit and the gambler had unlimited financial resources then the gambler would always win. His profit would be slightly less then 5 dollars for every 10 dollars wagered. Over the course of a 1000 bets with an initial $10 bet, doubled upon each loss, the player would win slightly less than $4800 profit. If the prison rule was implemented, the net win would be over $10000. There are two limits on the player that makes this outcome unlikely. First, there is virtually always a house limit on how much the player can bet and second, there is a practical limit to the amount of money the player can afford to gamble. Even billionaires might run out of money at some point. Either one of these limits by itself is sufficient to preclude a long-term win; most often both limits are present. An upper limit of $10000 might seem a remote possibility when starting with a $10 bet, however that limit would be reached after only 10 losses and ten consecutive losses should occur on average 16 times out of every 10000 bets. This probability is low, but it will happen eventually. Consequently, the player will usually lose in the long run. This is not necessarily true in a game such as blackjack or poker in which the player can affect the outcome by card counting or other similar strategies. However, in roulette, the eventual outcome is usually a loss.

The long-term outcome of playing roulette is a simple matter to compute based on the odds of winning. What is unclear however is how well a player using a doubling strategy would fair compared to a player betting the same amount each time, and how their success is affected by the size of the maximum bet. This paper reports on a simulation experiment that used the random number generator of QBASIC to simulate the outcome of betting on blacks, reds, odds or evens in which the pay out for a win was even money.

STUDY 1: DOUBLING VS. CONSTANT BETS

Method

A program was written by the author that used the random number generator from Microsoft's QBASIC to simulate the outcome of betting on blacks, red, odds or evens in which the payoff for a win was even money. The RND function was seeded using the computer's TIMER. The simulations were based on the American style roulette wheel in which there is a 0, a double 0, 18 red slots and 18 black slots. The program generated a random number to represent the slot on which the ball landed. If this matched one of the 18 predetermined slots (e.g., any red slot) then the simulated player won. If the number did not match one of the predetermined slots the player lost the bet. If the 0 or 00 occurred the bet was put into prison (see Wong, & Spector, 1996; Wagenaar, 1988 for descriptions of the rules of roulette). Money placed in prison was returned to the player, without profit if the player won on the next play. The computer looped until the player won. This loss/win cycle was used as the basis of length of play so that all simulated players in all conditions would end with a win regardless of the strategy used.

Three independent variables were manipulated: length of play (10 wins, 100 wins and 1000 wins), betting strategy (constant bet vs. doubling) and house maximum ($100 or $1000). The first bet in all cases was $10; in the constant bet condition, the amount of the bet never changed. The dependent variable was the player's net money.

Three hundred simulations were conducted in each combination of the independent variables resulting in a total of 3600 simulations. The probability of a win was 18 out of 38 for each trial. The probability of a loss was 18 out of 38. The probability of the stake being held in prison was 2 in 38. The probability of getting back a sum held in prison was 18/38. The house edge was 2.63%. It should be noted that the prison rule is available on some, but not all American roulette games (Wong & Spector, 1996, p. 74). This set of parameters was chosen as a compromise between the 1.35% house edge on the European wheel and the 5.26% house edge on the American wheel without the prison rule. With a different house edge the players' average losses would be different, but the pattern of results would be the same.

Results

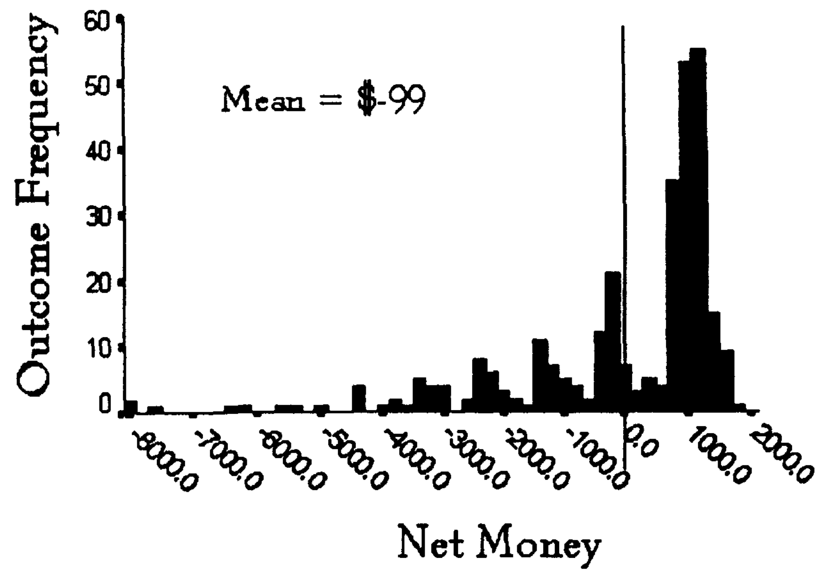
An analysis of variance was conducted on the data and found that every main effect and interaction was significant. Table 1 gives the means for each combination of these conditions. The distributions of outcomes for the constant bettors were roughly normal with a mean slightly below zero. The distributions for the doublers were negatively skewed with a mode slightly above zero for short duration of play, and below zero for longer duration of play. The lower tail of the distribution stretched down towards large negative numbers. In general doubling resulted in a greater loss than a constant bet, $F(1,3588) = 83.9$, $p < .01$. A gambler is likely to lose significantly more money if they doubled after losing. The longer a gambler plays, the more he or she will lose, $F(2,3588) = 187.2$, $p < .01$. A larger house limit, generally leads to a greater loss in the long run $F(1,3588) = 16.0$, $p < .01$. All interactions were significant. The largest interactive effect was the interaction of strategy with number of win cycles, $F(2, 3588) = 74.3$, $p < .01$. At the shortest interval, doublers (+$2.10) are better off than constant bettors (−$5.85). As the number of win loss cycles increases

Table 1
Average Net Money Comparing Constant Bets and Doubling

	House Maximum		*10 wins*	*100 wins*	*1000 wins*
Constant Bet	100	*M*	−5.5	−42.3	−560.6
		Mdn	0.0	−35.0	−550.0
		SD	48.8	145.5	473.2
	1000	*M*	−6.2	−45.7	−556.6
		Mdn	0.0	−30.0	−570.0
		SD	49.2	158.5	482.3
Doubling	100	*M*	−12.2	−157.2	−1528.37
		Mdn	+100.0	−110.0	−1525.0
		SD	182.6	593.1	1871.2
	1000	*M*	+16.1	−99.5	−3305.7
		Mdn	+100.0	+805.0	−2575.0
		SD	435.6	1752.9	6763.0

the doubling leads to a greater loss compared to a constant bet. The interaction between strategy and house upper limit, $F(1,3588) = 16.0$, $p < .01$) is due to an artifact in that a greater house limit made no difference to the constant bettors since the bet was never changed. The interaction between number of win cycles and house maximum bet, $F(2,3588) = 18.4$, $p < .01$, and the three way interaction, $F(2,3588) = 18.84$, $p < .01$, are due to the fact that the only condition under which the simulations estimated a net positive outcome (+$16) was the shortest time period (10 plays), with doubling and a house limit of $1000. Even after 100 win cycles the gamblers with the larger house limit (−$99.50) were better off than those with a smaller house limit (−$157.20). In contrast after 1000 win cycles a larger house limit (−$3305.73) lead to a larger average loss compared to a lower house limit (−$1528.37). In most contexts 300 simulations per condition is more than sufficient to give a very accurate estimate of a mean, however, with the house limit of 1000 and 10 win cycles, the standard deviation is very large. It should be noted that if the number of simulations was increased (e.g., 500) the means of all these cells would be negative and the average loss would be greater for the $1000 house limit condition in all cases, however the mode and median would still be positive. Consequently the mean is not particularly accu-

Figure 1
Distribution of New Money After 100 Loss/Win Cycles with Doubling and a House Limit of $1000 ($n$ = 300)



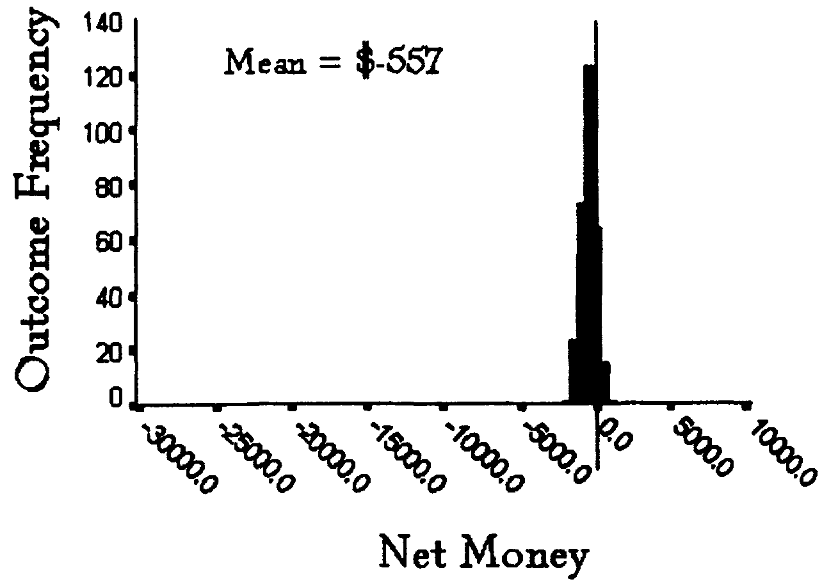
rate in this one cell. In the long run the mean will be negative, but because the mode is positive, the chances of getting a positive mean are fairly high. This particular cell nicely emphasises the difference between short-term outcome of this system where most people will win and long term outcome where most people will lose.

The benefits of doubling over a short period of time are shown in Figure 1. This graph depicts the distribution of net outcomes after 100 wins, when the house limit is $1000. The mean is a loss of $99, but the median and the mode are positive: most people using a doubling strategy for only 100 win cycles will win as is indicated by the median and mode. On average, however, they will lose as indicated by the mean.

These analyses were conducted assuming the player had unlimited resources. If a player was cut off after reaching the end of their 'bankroll', the average loss was actually less. This is due to the fact that by limiting the losses of some players the long tail of the skewed distribution was cut off resulting in a bimodal distribution.

Figures 2 and 3 show the long-term outcome for constant bettors

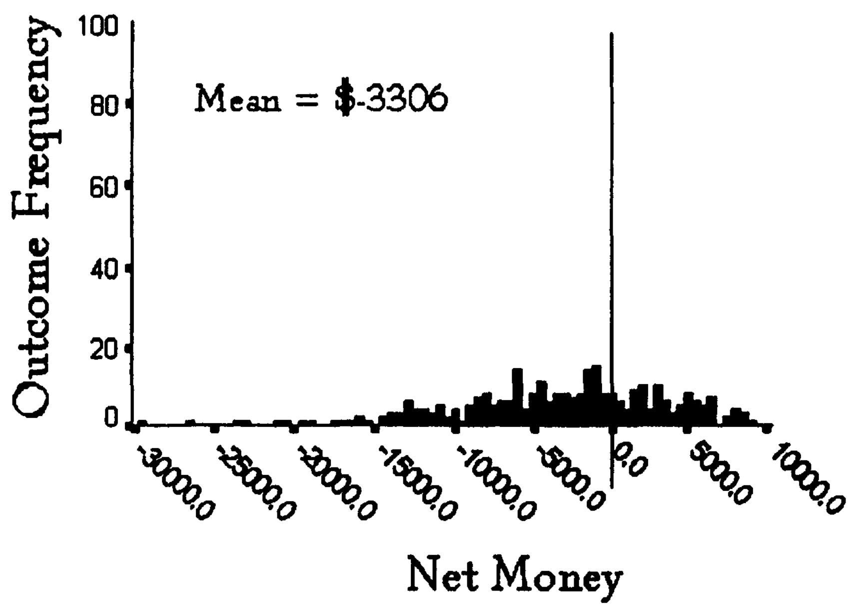
Figure 2
Distribution of New Money After 1000 Loss/Win Cycles with a Constant Bet (n = 300)



and doublers, respectively, after 1000 win cycles, and a maximum bet of $1000. More players using a doubling strategy have a net positive outcome and some players have won as much as $8000. However, the losses for doublers were as high as $30,000. In contrast the largest loss shown by a constant bettor is $1,900, and the largest win, a mere $900. The fact is that doubling can increase a players chances of winning and lead to a larger potential positive outcome, but in the long run, doubling results in a greater mean loss. As the number of loss/win cycles is increased the distribution of net money for doublers approaches a normal curve. The mean remains considerably lower for doublers than for constant bettors, and the standard deviation of outcome remains much greater than for constant bets.

Figure 4 shows the dramatic difference in the outcome between constant betting and doubling for an individual simulated player, with the same pattern of wins and losses. Four things are obvious from the figure. First, the general trend for both constant bets and doubling is a

Figure 3
Distribution of Net Money After 1000 Loss/Win Cycles with Doubling and a House Limit of $1000 ($n$ = 300)

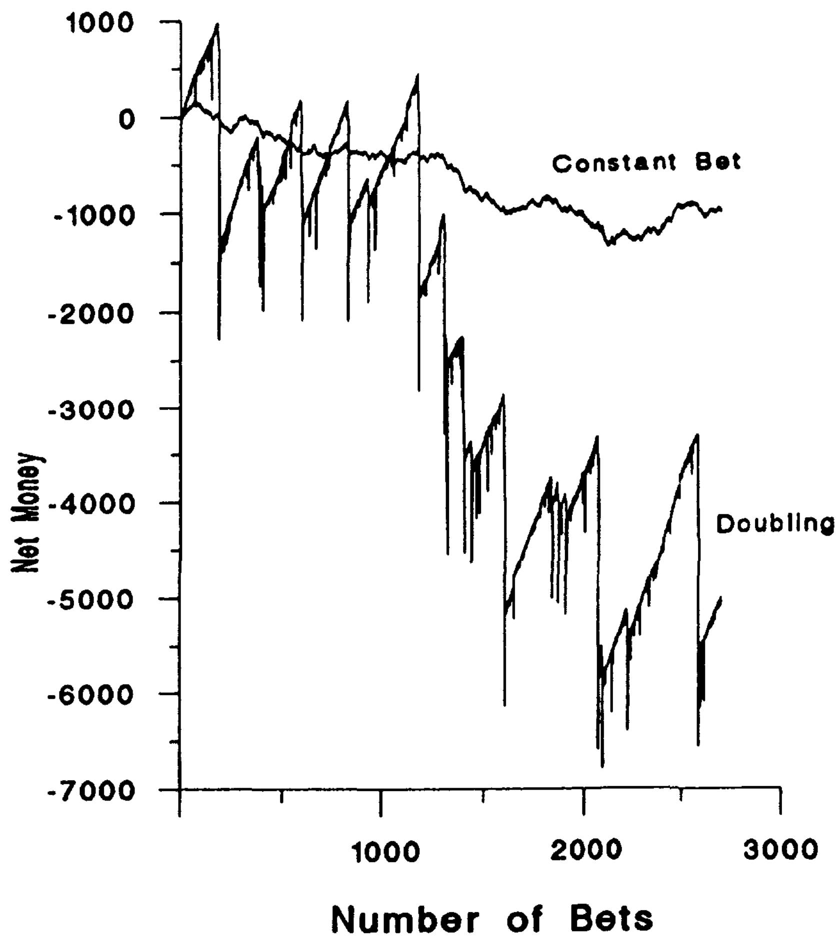


downward slope. Second, the variance of net money for the player that is doubling is much greater than the variance for the player that is betting a constant amount. Third, the doubler occasionally shows a net profit, and shows a profit more often than the constant bettor. Fourth, the slope of the doubler's net money is a zig zag with general upwards trends punctuated by rapid drops in net money as a result of those occasional losing streaks.

Discussion

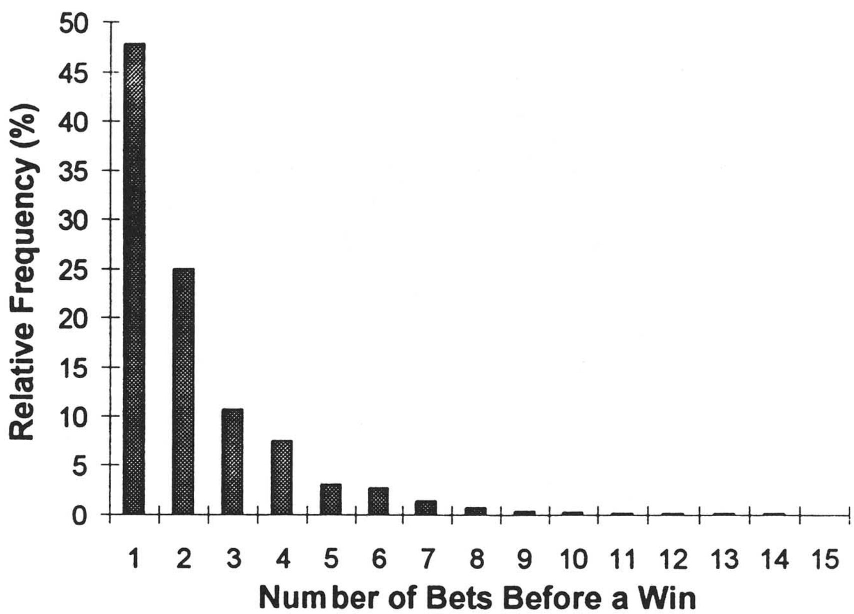
In general the results indicate that the more you play the more you lose. Doubling, a larger house limit and longer play all resulted in larger losses in the long run. In the short run, people using a doubling strategy are more likely to win. However, in the long term, they tend to lose. This contradiction occurs because the single most common outcome of a series of gambles ending in a win, is an immediate win

Figure 4
Comparison of Results of a Doubling vs. a Constant Bet Strategy for a Single Simulated Player



(p = 18/38) followed by losing once then winning (p = 20/38* 18/38), losing twice then winning (p = 20/38*20/38*18/38), and so on (Rachlin, 1990). The sequences are highly skewed with longer sequences occurring less frequently (see Figure 5). These long sequences of losses are rare, but occur often enough to virtually preclude the possibility of a doubler having a long-term positive outcome.

Figure 5
Relative Frequency of the Number of Times a Player Will Have to Gamble Before Winning



Although the outcome is initially skewed with most players winning in the short term, the distribution of net outcomes moves towards a normal distribution with the majority of players losing, as the number of loss/win cycles is increased. This is the expected long term result according to the central limit theorem which states that a distribution of averages will approximate normality regardless of the original distribution as the sample size increases (Herzberg, 1989).

Long losing sequences are rare, but devastating to the gambler using a doubling strategy. Even after 100 wins, while the median is positive, the mean is negative. A person that tries his luck with a doubling strategy will eventually run into a long series of losses. More to the point, one never knows when this will occur. A person that doubles until he wins once is most likely to take home his $10 profit. However, if he or she returns and tries this several times, he or she will eventually reach a long losing streak and lose everything that had been won.

The long term expected outcome of a system will most likely catch up to a system gambler whether their play is stretched out over many years, or concentrated into a single weekend.

A larger house limit resulted in greater long-term losses because more money was risked. Unless there is no house limit, the player using a doubling strategy will eventually lose; if however the upper limit was extremely large (e.g., a million times the minimum) and the player had unlimited financial resources, doubling would nearly guarantee a profit.

It should be noted that a minimum bet of $10 and a max of $1000 may not be very realistic, but it is possible in many casinos to work a Martingale system by switching between tables with different upper and lower limits. For example at one Casino I found one table with a upper limit of $50 on one floor and on another floor a table with a upper limit of $250. In this situation the gambler may not be as systematic as the simulated gamblers in this study, but the long-term results would be the similar.

Numerous variations on this doubling strategy were also implemented including a more conservative system in which bets were only increased by 1 unit upon each loss and a more liberal system in which bets were doubled plus an increase of one unit (the Big Martingale system). When compared to a constant bet strategy, all systems tested resulted in a larger standard deviation in outcome, a larger percentage of positive outcomes in the short run, and greater average losses in the long run. The riskier the system, the larger the percentage of simulated player with a net win in the short term, but the larger the net loss in the long term. As a general rule, systems involving increased bets result in greater average loss.

Some gamblers realise that doubling is a dangerous system, however, when combined with another system, such as looking for a hot table (e.g., a table with 8 more reds than blacks out of the last 20 spins of the wheel), they assume that doubling should then work if they bet on black. Although this system makes sense logically, this is not the way real random events work and therefore the doubling strategy is just as risky regardless of the number of blacks of reds that have appeared recently.

Another system popular with serious gamblers is to push their winning and increase their bets only after winning. This system also makes sense logically because when the gambler is risking a large amount of money they are risking their winnings rather than digging a deeper debt for themselves. But like other systems this too is doomed to fail. If the gambler doubles their bets when they win they will wipe out all their gains when they eventually do lose. As a result someone pushing his or her winnings, in a casino with a large house limit, might actually lose faster than someone using the Martingale system. A more conservative increase of say 1 unit after each win, will result in a more modest loss, but still a greater loss than compared to constantly betting the minimum.

Given the risks, why would anyone ever use a doubling or other incremental betting strategy? There is no simple answer. Part of the reason might be that the zigzagging ups and down of a doubling strategy is more exciting because it is more risky. Furthermore, a greater risk can lead to larger gains (Wagenaar, 1988). Roger Horbay of the Centre for Addiction and Mental Health (personal communications) calls Figure 4, the heartbeat of the devil because it so clearly illustrates the seductive and risky nature of a doubling strategy. In addition, since gamblers using a doubling strategy win most of the time, the intermittent reinforcement effects of the strategy may be very difficult to overcome.

Another possibility is that the naive human conception of random events is faulty. Consider a coin toss. If the coin is tossed randomly (that is no effort is made to manipulate it), on average heads will appear as often as tails. However, a series consisting entirely of heads is possible by chance alone. Randomness is clumpy (Gould, 1991). The naive human conception of random events, however, is that a random sequence looks random (see Kahneman & Tversky, 1982). A recent survey by the author found that most people believe that a loss is more likely after a win (Turner & Ialomiteanu, 1998, May). This false belief is sometimes called the 'law of averages' (Arnold, 1978). The reality is that neither a win, nor a loss affects the outcome of the next random event. This misconception about random events is akin to naive theories or mental models found in other domains such as physics (Gentner & Stevens, 1983) that results from a partial understanding of the concept. It is important however to note that reasoning with mental models is not irrational. In fact in some cases mental models are a very efficient means of reasoning (cf. Johnson-Laird, 1983). For example, people understand that a random series usually looks random. This is in fact true for most random sequences (Chaitin, 1975). However, they overgeneralize this rule and therefore systematically make errors (Kahneman & Tversky, 1982). Further, when confronted by a long sequence of wins or losses people make attribution errors (Wagenaar, 1988). A person that wins 10 times in a row might think they were having a hot streak or they might think that they have perfected an effective strategy. A person that loses 10 times in a row might attribute it to bad luck. In either case, because of these erroneous attributions, *the person would not reformulate their concept of random events.*

Suppose that the rules of probability matched this mental model and a person's chances of winning did indeed increased after losing, and their chances of losing increased after winning. Study two examined this possibility.

STUDY TWO: DOUBLING IN AN IDEAL WORLD

A second simulation was conducted (N = 300) to determine what would happen if random events were not completely independent. The program was modified so that each time a simulated player lost, their probability of winning was increased by 1/38 (e.g., from 18/38 to 19/38) and each time they won, the probability of winning was decreased by 1/38 (e.g., from 18/38 to 17/38). Simulations were conducted using a constant bet, doubling and a more 'conservative' strategy of incrementing the bet by 1 unit ($10) upon each loss. One hundred simulations were conducted per condition. The maximum bet was set at $100, and the number of loss/win cycles was 1000. Otherwise the procedure was identical to study 1.

Results

When the laws of probability were changed so that a player's chances of winning increased after each loss and decreased after each win, doubling became a much better strategy (M = $3741, SD = 1366) than constant bets (M = $-290, SD = 374). The more conservative strategy of increasing bets by 1 unit after each loss fell between these two (M = $1857, SD = 725). The differences between each of these three means were highly significant, $F(2,297) = 481.8$, $p < .01$. In this ideal world, the average number of bets before a win (2.09), was nearly identical to the first simulation in which the real laws of probability were used (2.10). However, even constant bettors faired better in this ideal world; because of the increased odds of winning after a loss, they were slightly more likely to get back money held in prison than were the constant bettors in study 1.

General Discussion

As expected, if the rules of probability matched the 'law of averages,' bettors using a doubling strategy or any other similar system would indeed prosper compared to those betting at a constant rate. It is argued that an incomplete mental model of random probability may be responsible for the use of doubling and similar systems in the real world. Mental models are an internal model or simulation that people use during visual perception, discourse comprehension, knowledge representation and reasoning (Eysenck, 1990). According to this theory, people reason using specific models of the world, rather than with generalised abstract rules. Mental models tend to be specific and concrete and consequently do not usually represent all possible conditions. Johnson-Laird (1983) found a direct relationship between the number and complexity of mental models needed to solve a problem and the number of reasoning errors that people make on a problem. To fully appreciate random events you would need to understand all possible sequences. It is argued that most people have an incomplete mental model of random sequences. This model is based on the accurate perception that random events of equal probability tend to alternate erratically (e.g., H-T-H-H-T-H-T-T) and that sequences that 'look' random are more common than sequences that do not 'look' random. However, this model does not include the abstract notion that all specific sequences occur with equal probability regardless of whether they 'look' random or not. Consistent with the view, younger children are less likely than older children to prefer 'random' tickets suggesting that a person's understanding of what random numbers should look like increases with age (Derevensky, Gupta & Herman, 1997, June). Acquisition of the 'representative heuristic' is the results of a better, but still incomplete, understanding of random events.

There are important implications of this theory. First, many gamblers in treatment claim to have considerable skill as gamblers (Toneatto, 1998) and often report having long winning periods. Gamblers relying on doubling with a generous house upper limit, will go home with a profit more often than not, perhaps giving the gambler a false belief in his or her level of skill or the effectiveness of the strategy used. The results of this simulation may help explain why such 'skilled' gamblers end up in treatment after suffering enormous losses. Second, other systems may be equally suspect. For example, many gamblers believe that they can spot a biased wheel (Wong & Spector, 1996). However, since randomness is clumpy, an apparent bias is likely to occur quite frequently. This is not to say that biased wheels do not exist (see Bass, 1985), but rather that many apparently biased wheels might be type 1 errors. The probability of finding a wheel that has a statistically significant bias that is not really there is 1 out of 20; if a casino includes several roulette wheels, there is a very high probability that at least 1 will exhibit a significant bias that is not really there. Given the difficulty that most people have with statistical reasoning (Kahneman & Tversky, 1982), it is suggested that searching for a bias is more likely to lead to losses rather than wins. Third, viewing excessive gambling as a rational result of an incomplete mental model suggests that helping a person develop a more complete mental model of probability may be an important step (but not the only step) towards solving the person's gambling problem.

These results also have significant clinical implications. It is very important for clinicians to understand the fine details of systems and strategies that gamblers use in order to help the gambler correct their errors in reasoning. If a gambler is using the Martingale system it is important to help him or her understand that in the long run it just simply can't work. He or she may have experienced huge or frequent wins and thus may hold to the belief that if they could just learn to anticipate the sudden turn around, they would now be up thousands of dollars. Showing a gambler the pattern in Figure 4 may come as quite a revelation to them. The expression "that's exactly what happened to me" is not uncommon (Horbay, R., personal communications). The gambler can quickly understand the illusion of the winning streaks. The gambler may have been seeking out the rush and exhilaration of such winning streaks without realising the long-term outcomes must be negative. Clinicians need to understand the potency of such winning streaks and how gamblers can win the majority of the time but still end up losing. Simulations of the sort used here have potential for use in treatment, but caution must be exercised. These systems do work sometimes and the gambler might pick up the wrong message. Upon seeing Figure 1, more than one person (including counsellors) have remarked, "I want to try that system." So, the emphasis needs to be not on the winning streaks but on the final outcomes.

Furthermore it might help in the development of a therapeutic relationship between counsellor and client, if the counsellor tells the gamblers that they're not irrational, but in fact have shown some ability in logic. Being told that their system is logical, but based on an incomplete model of reality, might leave the gambler with an improved self-esteem. The counsellor can then help the clients see how they can use of their logical ability in other areas outside of gambling. Clinician also need to realise that not helping the client correct their erroneous beliefs in their system will inevitably become a relapse issue even if the client is successful in initiating behavioral change. Clients may reduce or stop their gambling as a result of negative consequences to finances, family or careers. However, they may still harbour a belief in their winning system long after other pressures are lifted. If the belief "I can win" is not put in its proper context, the gambler may again resort to playing their system, repeating the disastrous long-term outcome.

In addition, it is important for the gambling counsellor to understand the flaws in the systems that the gambler is using because there is a danger that the counsellor might begin to believe that the system could work and be tempted to try the system. After all, the systems, on the surface, may sound very logical. Therefore, it is extremely important for clinicians to have a complete mental model of random probability.

It is unclear to what extent these findings can be generalised to other forms of gambling. Roulette is a system players dream due to the vast variety of possible betting strategies (Arnold, 1978). Other games like slot machines, black jack and racetrack betting attract different kinds of systems. However, it is not unusual for problem gamblers to double down when losing, winning or chasing on various games. Furthermore it is likely that different types of people play games for different reasons. In this paper I have only examined one particular type of gambler, the system player. It is argued that the mental model view of a gambler's concept of random probability can also make sense of other games and other types of gamblers as well, however, more study, including a detailed examination of the systems' used, is needed.

In conclusion it is suggested that an incomplete mental model of random probability, may be in part responsible for the enormous losses that problem and pathological gamblers often experience. A doubling strategy would in fact work if randomness matched the human mental model of what randomness should look like. Sadly, randomness does not, and doubling therefore in the end results in greater losses, not a guaranteed win. Understanding, a gambler's system and illuminating its' flaws might be an important part of the treatment of a problem gambler.

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